td ST4.0

N°243

(1) $\boxed{\overrightarrow{OG} = \frac{a}{3}\vec{x} + \frac{h}{3}\vec{z}}$

$\boxed{M = \rho_b \frac{h \cdot a}{2} \cdot l}$

$M = 2600 \frac{30 \cdot 20}{2} 80$

$M = 62{,}4\ 10^3$ tonnes (10^6 kg)

(2) $\boxed{\{T_{p\to b}\}_G = \begin{Bmatrix} 0 & 0 \\ 0 & 0 \\ -Mg & 0 \end{Bmatrix}_{G,R}}$

Varignon: $\overrightarrow{\mathcal{M}_{O,p\to b}} = \underbrace{\overrightarrow{\mathcal{M}_{G,p\to b}}}_{\vec{0}} + \overrightarrow{OG} \wedge \overrightarrow{R_{p\to b}}$

$= \left(\frac{a}{3}\vec{x} + \frac{h}{3}\vec{z}\right) \wedge -Mg\vec{z}$

$\begin{cases} \vec{x} \wedge \vec{z} = -\vec{y} \\ \vec{z} \wedge \vec{z} = \vec{0} \end{cases}$

$\overrightarrow{\mathcal{M}_{O,p\to b}} = \frac{a}{3} Mg\, \vec{y}$

d'où $\boxed{\{T_{p\to b}\} = \begin{Bmatrix} 0 & 0 \\ 0 & \frac{a}{3}Mg \\ -Mg & 0 \end{Bmatrix}_{O,R}}$

(3) $\{T_{e\to b}\} = \begin{Bmatrix} \overrightarrow{R_{e\to b}} = \int_0^h \lambda(z)\, \vec{x}\, dz \\ \overrightarrow{\mathcal{M}_{O,e\to b}} = \int_0^h \overrightarrow{OQ} \wedge \lambda(z)\vec{x}\, dz \end{Bmatrix}_O$

- $\overrightarrow{R_{e\to b}} = \int_0^h \rho_e g l (h-z) + p_0 l \; dz\, \vec{x}$

$= \left[\rho_e g l \left(hz - \frac{z^2}{2}\right) + p_0 l z\right]_0^h \vec{x}$

$= \left(\rho_e g l \left(h^2 - \frac{h^2}{2}\right) + p_0 l z\right) \vec{x}$

$\boxed{\overrightarrow{R_{e\to b}} = \left(\rho_e g l \frac{h^2}{2} + p_0 l h\right)\vec{x}}$

- $\overrightarrow{\mathcal{M}_{O,e\to b}} = \int_0^h z\vec{z} \wedge (\rho_e g l (h-z) + p_0 l)\vec{x}\, dz$

avec $\vec{z} \wedge \vec{x} = \vec{y}$

$= \int_0^h \rho_e g l (hz - z^2) + p_0 l z \; dz \cdot \vec{y}$

$= \left[\rho_e g l \left(h\frac{z^2}{2} - \frac{z^3}{3}\right) + p_0 l \frac{z^2}{2}\right]_0^h \vec{y}$

$= \left(\rho_e g l \left(\frac{h^3}{2} - \frac{h^3}{3}\right) + p_0 l \frac{h^2}{2}\right)\vec{y}$

$\overrightarrow{\mathcal{M}_{O,e\to b}} = \left(\rho_e g l \frac{h^3}{6} + p_0 l\, h^2/2\right)\vec{y}$

D'où $\{T_{e\to b}\} = \left\{\begin{matrix} X_{eb} & 0 \\ 0 & M_{eb} \\ 0 & 0 \end{matrix}\right\}_{O,R}$

avec $\boxed{X_{eb} = \rho_e g l \frac{h^2}{2} + p_0 l h}$

$X_{eb} = 1000.10.80.\frac{30^2}{2} + 10^5.80.30$

$\underline{X_{eb} = 600 MN}$

$\boxed{M_{eb} = \rho_e g l \frac{h^3}{6} + p_0 l \frac{h^2}{2}}$

$M_{eb} = 1000.10.80.\frac{30^3}{6} + 10^5.80.\frac{30^2}{2}$

$\underline{M_{eb} = 18 GNm}$

(4) Equilibre du barrage : $\{T_{p\to b}\}_O + \{T_{e\to b}\}_O + \{T_{o\to b}\}_O = \{0\}$

$\left\{\begin{matrix} 0 & 0 \\ 0 & \frac{a}{3}Mg \\ -Mg & 0 \end{matrix}\right\}_{O,R} + \left\{\begin{matrix} X_{eb} & 0 \\ 0 & M_{eb} \\ 0 & 0 \end{matrix}\right\}_{O,R} + \left\{\begin{matrix} X_{ob} & 0 \\ 0 & M_{ob} \\ Z_{ob} & 0 \end{matrix}\right\}_{O,R} = \{0\}$

pb plan $(O, \vec{x}, \vec{z})$

d'où $\boxed{X_{ob} = -X_{eb}} = -600 MN$

$\boxed{Z_{ob} = Mg} = 62,4.10^6.10 = \underline{624 MN}$

$\boxed{M_{ob} = -M_{eb} - \frac{a}{3}Mg}$ $M_{ob} = -18 - \frac{20}{3}.62,4.10^6.10$

$\underline{M_{ob} = -22 GNm}$

(5) Conclusion : le cahier des charges est respecté car
- $|X_{ob}| = 600 MN \simeq 600 MN$
- $|X_{ob}| = \underline{600 MN} < Mg = \underline{624 MN}$