13h10 td AL 1 3

① $\langle u_1 \rangle = \dfrac{1}{2\pi}\int_0^{2\pi} u_1(\theta)\,d\theta = \dfrac{1}{2\pi}\int_0^{\pi} U_m \sin(\theta)\,d\theta$ car $u_1(\theta)=0$ entre π et 2π

avec $\theta = \omega t$

$$\langle u_1 \rangle = \frac{U_m}{2\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{U_m}{2\pi}\Big(\underbrace{-\cos(\pi)}_{1} - \underbrace{(-\cos(0))}_{1}\Big)$$

$$\langle u_1 \rangle = \frac{U_m}{\pi}$$

Valeur efficace $U_1 = \sqrt{\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} u_1^2(\theta)\,d\theta} = \sqrt{\dfrac{1}{2\pi}\displaystyle\int_0^{\pi} U_m^2 \sin^2(\theta)\,d\theta}$

car $u_1(\theta)=0$ entre π et 2π

Linéarisation de $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

d'où $U_1 = \sqrt{\dfrac{U_m^2}{2\pi}\displaystyle\int_0^{\pi} \dfrac{1}{2} - \dfrac{\cos 2\theta}{2}\,d\theta} = \sqrt{\dfrac{U_m^2}{2\pi}\left[\dfrac{\theta}{2} - \dfrac{\sin 2\theta}{4}\right]_0^{\pi}}$

$$U_1 = \sqrt{\frac{U_m^2}{2\pi}\left(\frac{\pi}{2} - \cancel{\frac{\sin 2\pi}{2}} - \left(\frac{0}{2} - \frac{\sin 0}{2}\right)\right)}$$

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$$U_1 = \frac{U_m}{2}$$

② Valeur moyenne $\langle u_2 \rangle = \dfrac{1}{\pi}\displaystyle\int_0^{\pi} u_2(\theta)\,d\theta = \dfrac{1}{\pi}\displaystyle\int_0^{\pi} U_m \sin(\theta)\,d\theta$ (période π pour $u_2(\theta)$)

$$= \frac{1}{\pi}\left[-U_m \cos(\theta)\right]_0^{\pi} = \frac{1}{\pi}\Big(\underbrace{-U_m\cos(\pi)}_{+U_m} - \underbrace{-U_m\cos(0)}_{+U_m}\Big)$$

$$\langle u_2 \rangle = \frac{2}{\pi}U_m$$

Valeur efficace $U_2 = \sqrt{\dfrac{1}{\pi}\displaystyle\int_0^{\pi} u_2^2(\theta)\,d\theta} = \sqrt{\dfrac{1}{\pi}\displaystyle\int_0^{\pi} U_m^2 \sin^2(\theta)\,d\theta}$

linéarisation : $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

d'où $U_2 = \sqrt{\dfrac{U_m^2}{2\pi}\displaystyle\int_0^{\pi}(1-\cos 2\theta)\,d\theta} = \sqrt{\dfrac{U_m^2}{2\pi}\left[\theta - \dfrac{\sin 2\theta}{2}\right]_0^{\pi}}$

13h40 $U_2 = \sqrt{\dfrac{U_m^2}{2\pi}\left[\pi - \dfrac{\sin 2\pi}{2} - \left(0 - \dfrac{\sin 0}{2}\right)\right]}$ $U_2 = \dfrac{U_m}{\sqrt{2}}$

③ $\langle u_3 \rangle = \dfrac{1}{T}\displaystyle\int_0^{T} u_3(t)\,dt = \dfrac{1}{T}\left(\int_0^{T/2} E\,dt + \int_{T/2}^{T} -E\,dt\right)$

$\langle u_3 \rangle = 0$ car aire positive = - aire négative

$U_3 = \sqrt{\dfrac{1}{T}\displaystyle\int_0^{T} u_3^2(t)\,dt} = \sqrt{\dfrac{1}{T}\displaystyle\int_0^{T} E^2\,dt} = \sqrt{\dfrac{E^2}{T}\left[t\right]_0^{T}}$

$$U_3 = E$$

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